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LETTER TO THE EDITOR

Polymers with a bimodal disorder distribution and directed percolation

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Abstract. The ground-state scaling properties of directed paths on a $(1+1)$-dimensional lattice are reanalysed. To each bond energy 0 or 1 is randomly assigned with probability p or $1-p$, respectively. At variance with previous claims, the result strongly suggests that only one universality class exists for $0 < p < 1$, except for $p = p_c$, the directed percolation threshold.

Directed polymers (DPs) in random media [1–3] have been one of the major topics in the study of disordered systems in the last decade: the universality class of the ground state has been established in the weak disorder limit, as well as its connections in $1+1$ space-time dimensions with other problems such as domain walls in random ferromagnets [3, 4], the dynamics of growing interfaces governed by the Kardar–Parisi–Zhang (KPZ) equation [3, 5] and the Burgers equation of fluid motion [6]. Behind the weak disorder universality class there is always the assumption that the disorder distribution becomes Gaussian in some hypothetical renormalization group (RG) procedure. However, recently Zhang and Lebedev [7] devoted some attention to the effects of a bimodal distribution of disorder on the universality class of DPs. In their model each bond b of a lattice is randomly assigned an energy $E_b = 0$ (with probability p) or $E_b = 1$ (with probability $1-p$). In the ground state, to each walk W an energy E_W is given according to the rule:

$$E_W = \sum_{b \in W} E_b \tag{1}$$

where the sum is over the bonds visited by the path.

On the basis of some numerics, they proposed the interesting conjecture, that there is a single universality class, different from the weak disorder one, in the whole interval $0 < p \leqslant p_c$, where p_c is the directed percolation threshold (the 0 energies percolate through the lattice).

However, invoking universality and using more refined numerical results, we show that, apart from the single case $p = p_c$, DPs are still in the weak disorder universality class characterized by the exponents $\zeta = \frac{2}{3}$ for the transverse wandering fluctuations and $\omega = \frac{1}{3}$ for the ground-state energy fluctuations.

If $p = p_c$ the incipient percolating cluster (IPC) is a fractal, and the polymers share the same wandering properties of the backbone[7], with $\zeta_p = 0.6326 \pm 0.0002$ [8]. If $p \geqslant p_c$, the polymers lay on the directed percolation cluster, avoiding all the 1-bonds and there are no energy fluctuations associated with them whereas the entropy, the logarithm of the number of walks with one extremum fixed, fluctuates with the ω exponent. This is the case studied in [9, 10] where it was shown that the exponents remain the same as in the weak disorder problem [1]. Below p_c there is no infinite cluster spanning the lattice, and the polymers necessarily pick up some bonds with energy equal to 1. Lebedev and Zhang [7] numerically found that the directed percolation universality class also extends for $0 < p < p_c$. They tried to justify this conclusion by arguing that below p_c the polymers still see the IPC: the 1-bonds that break its connectedness can be seen as a disorder on it. Indeed it was numerically shown that polymers forced to live on the IPC, disordered by a random energy on the allowed bonds, have $\zeta = \zeta_p$ [9, 10].

On the other hand, a careful renormalization approach (which can be made exact on hierarchical lattices) shows that if $p < p_c$ the probability of finding an infinite connected path of 0-bonds vanishes, increasing the length of the lattice (or, which is somehow equivalent, performing some coarse graining step). Therefore on long scales the IPC disappears completely, and the polymers are simply subject to a disordered energy landscape where the whole lattice is seen by the DP, and not only the IPC. Therefore, based on this argument, at variance with [7], we expect to find the weak universality class when $p < p_c$.

This is consistent with [11], where computations on hierarchical lattices of any (effective) dimension using a bimodal disorder distribution (taking care that the lower energy do not percolate) have been compared with the $(1+\epsilon)$-dimensional expansion with a Gaussian distribution, showing a perfect agreement.

In order to set out the controversy, we have performed simulations on a square lattice for times t up to 2^{13} and taking averages over 10 000 realization of the disorder (in general richer than the statistics in [7]).

The computational technique is a transfer matrix approach to the problem, on a directed square lattice as in figure 1 where a polymer starting in O does not feel any finite-size effect. Every bond is assigned an energy that can be either 0 or 1 with probabilities p and $1-p$ respectively. The energy configuration at time t is obtained from the configuration at time $t-1$ according to the rule

$$E_k(t) = \min[E_i(t-1) + \epsilon_{ik}, E_j(t-1) + \epsilon_{jk}] \tag{2}$$

where i and j are the sites at time $t-1$ from which site k at time t can be reached, $\epsilon_{ik}(\epsilon_{jk})$

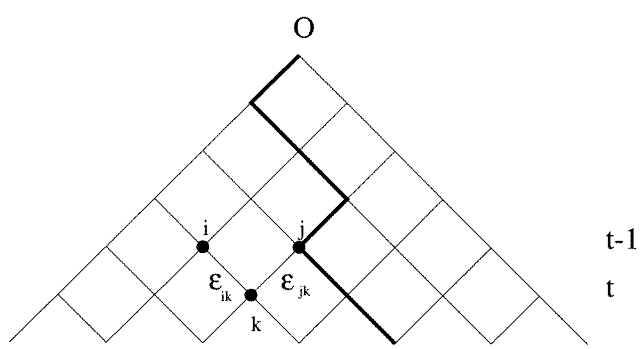

Figure 1. Directed lattice used in the simulation. Polymers are chosen with an extremum fixed in O.

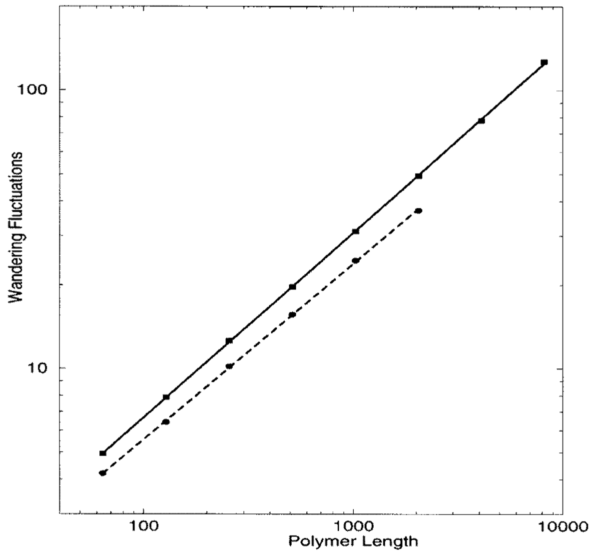

Figure 2. Wandering of directed polymers for $p = 0.3 < p_c$ (squares) and $p = p_c$ (circles). Error bars are smaller than the symbols. The wandering exponents are $\zeta = 0.666 \pm 0.002$ ($p = 0.3$) and $\zeta = 0.633 \pm 0.003$ ($p = p_c$) and correspond to the slopes of the shown straight lines.

is the energy of the bond connecting sites i (j) and k. Eventually, the minimum energy is chosen from the last row (the final time).

The results are shown in figure 2. Data for $p = 0.3 < p_c$ clearly show the weak disorder universality class with a wandering exponent $\zeta = 0.666 \pm 0.002$, whereas for $p = p_c = 0.6447$ the wandering exponent is 0.633 ± 0.003, very close and compatible with ζ_p.

These results must be compared with those of [7], where the results for $p = p_c$ and $p = 0.35$ are shown and those for $p = 0.02$ are mentioned. Whereas we both agree on the result for $p = p_c$, our data clearly indicate that for $p = 0.3$ (close to their value $p = 0.35$) the universality class is the weak disorder one. This result excludes the possibility that the directed percolation universality class, holding at $p = p_c$, extends down to $p = 0.02$ as stated in [7]. On the other hand, it does not exclude the presence of another critical point $0.3 < p_t < p_c$ such that the universality class is the weak disorder one if $p < p_t$ and the directed percolation one if $p_t < p < p_c$. Simulations for values of p closer to p_c are unfortunately quite difficult because they are affected by very strong crossover effects and therefore they cannot be conclusive. However, on the basis of the previously mentioned theoretical argument we believe that the existence of such peculiar critical point p_t is quite implausible. To corroborate this conclusion we have directly studied the Lebedev–Zhang model on a hierarchical lattice (the one drawn in figure 3). In such a lattice the exponent ω (ζ is not defined) can be obtained numerically by applying RG iterations for the probability distribution [10, 11]. In figure 4 the values of ω that we obtained are plotted versus the number of RG iterations for three different values of p (0.1, 0.3, 0.49), which are all below the percolation threshold for the lattice $p_c = 0.5$. For all the three cases, as the number of iterations increases, ω approaches the value 0.3, which is the one expected for the universality class of DPs in weak disorder on the hierarchical lattices [10, 11]. These results again seem to enforce the existence of only one universality class for all the region $p < p_c$.

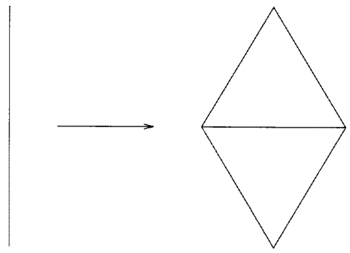

Figure 3. Iterative construction of the hierarchical lattice used for the numerical RG. The percolation threshold for this lattice is $p = 0.5$.

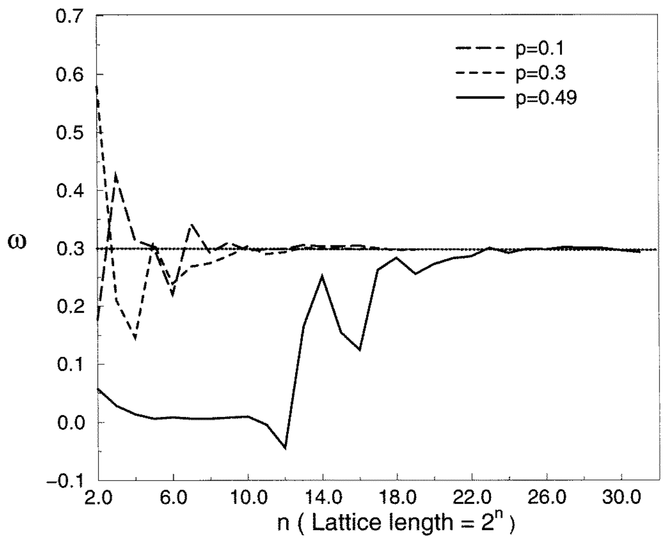

Figure 4. Ground-state energy fluctuation exponent on the hierarchical lattice as in figure 3, as a function of RG iterations n (the length of the lattice is 2^n). Three different values of p are considered, as from the legend. The horizontal line represents the value 0.30 ± 0.01 obtained in the weak disorder case in [11].

In conclusion, on the basis of numerical simulations and RG arguments, we believe that DPs with a bimodal distribution of energies are in the same universality class of weak disorder for all the region $p < p_c$.

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